5/29/25

$\omega^0_F(\overset{+}{\circ}) = \overset{+}{F}(\circ)$

$\Sigma F(\text{head}) - F(\text{tail}) = F(b) - F(a)$

$=$

$\underbrace{DF(\circ)}_{\text{matrix}} \underbrace{(\text{head}-\text{tail})}_{\text{vector}}$

$\omega^0_f(\overset{+}{\circ}) = \pm f(\circ)$

"sum over the bdry" (ext. der.)

$\omega^{1,0}_f(\nearrow) = \sum_{\circ \text{ of bdry of } \nearrow} \omega^0_f(\overset{+}{\circ})$

$= f(\text{head}) - f(\text{tail})$

FTC (take $\int_C$)

$\int_C \vec{\nabla} f \cdot d\vec{r} = f(b) - f(a)$

$\nabla f(\circ) \cdot \vec{dr}$

generalize

$\omega^{0,1}_f = \omega^1_{\nabla f}$, $\overset{+}{F}$

$\omega^1_F(\nearrow) = F(\circ) \cdot \nearrow$

$\omega^{2,(1,0)}_f(\square) = \sum_{\nearrow \text{ of bdry of } \square} \omega^{1,0}_f(\nearrow) = 0$

$\text{curl}\,\nabla f(\circ) \cdot \vec{dS}(\square)$

$\Rightarrow \text{curl}\,\nabla f = 0$

$\omega^{2,1}_f(\square) = \sum_{\nearrow \text{ of bdry of } \square} \omega^1_f(\nearrow)$

if 2D seg in $\mathbb{R}^2$, $\rightarrow \underbrace{\text{curl}\,F(\circ)}_{\text{scalar}} \underbrace{dA(\square)}_{\text{scalar}}$ ($\pm$ ← orientation here, not really; since clockwise vs. counterclockwise impact the limit → curl.)

FTC $\iint_S$

$\iint_{R \text{ in } \mathbb{R}^2} \text{curl}(F)(x,y)\,dA = \oint_{\partial R} \vec{F} \cdot d\vec{r}$

$\iint_{S \text{ in } \mathbb{R}^3} \text{curl}F \cdot \vec{dS} = \oint_{\partial R} \vec{F} \cdot \vec{dr}$

if 2D seg in $\mathbb{R}^3$, $\rightarrow \underbrace{\text{curl}\,F(\circ)}_{\text{vec}} \cdot \underbrace{\vec{dS}(\square)}_{\text{vec}}$

generalize

$\omega^{2,1}_F = \omega^2_{\text{curl}F}$, G

$\omega^2_G(\square) := G(\circ) \cdot \underbrace{\text{normal}(\square)}_{= \text{cross}(\square)}$

(by manipulating bivectors)

$\omega^{3,2}_G(\square) := \sum_{\square \text{ of bdry of } \square} \omega^2_G(\square)$

$\iiint_V \text{div}\,G(\circ)\,dV = \oiint_S G \cdot \vec{dS}$

orientations compatible via RH rule

FTC $\iiint_V$

$\omega^{3,(2,1)}_F(\square) := \sum_{\square \text{ of bdry of } \square} \omega^{2,1}_F(\square) = 0$

$\pm \text{div}\,G(\circ)\,dV(\square)$ (depend on out/in normal)

gen. $\omega^{3,2}_G = \omega^3_{\text{div}\,G}$

$\omega^3_g(\square) = \pm g(\circ)\,dV(\square)$

$= g(\circ)\,\det(\square)$

$\pm \text{div}\,G(\circ)\,dV(\square)$

$\Rightarrow \text{div}\,\underbrace{\text{curl}F}_{G} = 0$

Stop b/c in $\mathbb{R}^2$, no way to eat 4D segments.